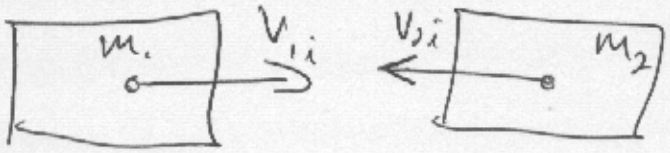
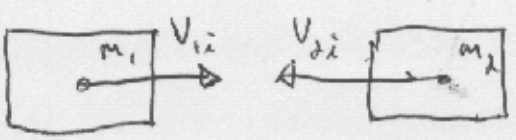
Finding the new velocities of two objects after they collide.



Jobe Makar

Before



Momentum

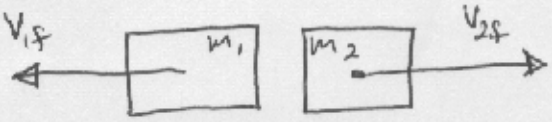
$P_{1i} = m_1 V_{1i}$, $P_{2i} = m_2 V_{2i}$

$P_i = P_{1i} + P_{2i}$

Kinetic Energy

$KE_i = \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2$

After



momentum

$P_{1f} = m_1 V_{1f}$, $P_{2f} = m_2 V_{2f}$

$P_f = P_{1f} + P_{2f}$

Kinetic Energy

$KE_f = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

The goal: To find V_{1f} and V_{2f}

$P_i = P_f \Rightarrow$ Conservation of momentum

$$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f} \quad \boxed{0}$$

$$\Rightarrow m_1 (V_{1i} - V_{1f}) = m_2 (V_{2f} - V_{2i}) \quad \boxed{1}$$

$KE_i = KE_f \Rightarrow \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

cancel $(\frac{1}{2})$ terms and group by mass

$$m_1 (V_{1i}^2 - V_{1f}^2) = m_2 (V_{2f}^2 - V_{2i}^2)$$

factor it

$$m_1 (V_{1i} + V_{1f})(V_{1i} - V_{1f}) = m_2 (V_{2f} + V_{2i})(V_{2f} - V_{2i}) \quad \boxed{2}$$

divide $\boxed{2}$ by $\boxed{1}$

$$V_{1i} + V_{1f} = V_{2f} + V_{2i} \quad \boxed{3}$$

with $\boxed{0}$ + $\boxed{3}$ we can easily find the new velocities

rewriting $\boxed{3}$ we get $V_{1i} - V_{2i} = V_{2f} - V_{1f}$

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$P = P_i$

$V = V_{1i} - V_{2i}$

$P = m_1 V_{1f} + m_2 V_{2f}$ ④

$V = V_{2f} - V_{1f} \Rightarrow \boxed{V_{1f} = V_{2f} - V}$

Sticking this in this

$P = m_1 (V_{2f} - V) + m_2 V_{2f}$

$P = V_{2f}(m_1 + m_2) - m_1 V$ ●

$$\boxed{V_{2f} = \frac{P + m_1 V}{m_1 + m_2}}$$

The result

Once this is calculated, stick V_{2f} back into either [0] or [3] to get V_{1f}